

Study of Distortions in Statistics of Counts in CCD Observations using the Fano Factor

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Abstract. Factors distorting the statistics of photocounts when detecting objects with low fluxes were considered here. Measurements of the Fano factor for existing CCD systems were conducted. The study allows one to conclude on the quality of the CCD video signal processing channel. The optimal strategy for faint object observations was suggested.

1. INTRODUCTION

There is an opinion formed among astronomers that charge-coupled array devices are almost ideal light detectors which do not distort the input Poisson signal, and observations with them are restricted only by the statistics of the original flux.

The distribution of counts during image acquisition on the whole differs from the distribution of photons that causes it. It is conditioned by many factors:

- readout noises;
- inhomogeneity of the array sensitivity;
- instability and non-linearity of the transfer function;
- cosmic ray traces and interference effects on thinned sensors (fringes).

If it refers to faint object observations on a relatively bright sky background, then the list of factors can be completed with correct background subtraction which also distorts the statistics of photocounts and, consequently, lowers the signal-to-noise (S/N) ratio of the final result.

The aim of the present paper is to reveal restrictions imposed by the above factors on low-flux objects acquisition with CCDs.

2. DISTORTIONS OF CCD COUNTS' STATISTICS

As a rule, when operating with CCD a hypothesis is accepted that the statistics of output counts follows the Poisson law (Howell, 2006). This very reason is used in determination of the gain factor (*gain*) of an analog-to-digital converter in the CCD readout channel. Usually a

sequence of image pairs of uniformly illuminated field with different exposures t is used. In this case, the measured dispersion of sequence of counts in the image difference $\Delta I(x_i, y_i, t)$ is determined by the relation:

$$D_I(t) = (\overline{\Delta I(t)^2} - \overline{\Delta I(t)})^2/2. \quad (1)$$

In the case of the Poisson law, the dispersion of counts will be proportional to the mean value:

$$D_I(t) = \text{gain} \times \overline{I(t)}. \quad (2)$$

Naturally, realistic dependences of dispersion on the mean differ from the linear ones. At small counts—as a consequence of readout noise, at greater—due to non-linearity of the transfer function of the CCD readout channel. It is known that one of the strongest testing criteria of the Poisson distribution of counts is connected with the study of the so-called Fano factor (Fano, 1947) (dispersion index, variation factor (Cox & Lewis, 1966)) $k(t)$ which is the relation of dispersion to the mean value:

$$k(t) = D_I(t)/\overline{I(t)}. \quad (3)$$

For the Poisson distribution, $k(t) \equiv 1$. To check the difference between the CCD and Poisson distributions of counts, we measured the Fano factor for existing dCCD systems (Markelov et al., 2000; Afanasieva, 2015) designed by the Advanced Design Laboratory and operated at the 6-m SAO RAS telescope at present. Figure 1 shows the measurements for the CCD readout channel with EEV CCD42-90 detector.

As can be seen from Fig. 1, the differences of the Poisson distribution are most noticeable at low fluxes and are determined by readout noise and bandwidth of a video channel:

$$k(t) \approx RON^2/\overline{I(t)}. \quad (4)$$

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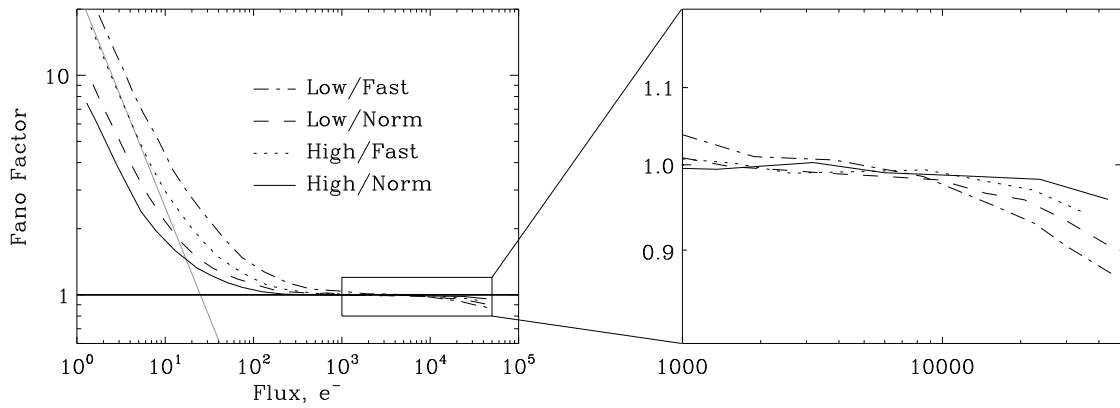


Fig. 1. Dependence of the Fano factor in the flat field image on the average flux for various operation modes of the CCD42-90 readout channel: *Low* and *High* are *gain* values equal to $1.82e^-/\text{ADU}$ and $0.48e^-/\text{ADU}$ respectively; *Fast* and *Norm* are readout rates, 400 and 100 Kpixel s^{-1} respectively. An oblique grey straight line shows the asymptotic behavior in the region of low fluxes for the *High/Fast* mode with readout noise (RON) $5e^-$.

Inconsiderable fall of the Fano factor in the greater-flux region appears due to the dispersion decrease which is discussed in recent works (Downing et al., 2006; Ma et al., 2014; Guyonnet et al., 2015) and is probably associated with a charge redistribution between pixels. This effect is one more factor limiting accuracy at high levels of signal in pixel.

Thus, one can conclude the fact that the count statistics in certain pixels of a realistic CCD is close to the Poisson one in a relatively wide range of signal intensity.

However, in spite of the fact that the CCD detector system does not practically distort the Poisson statistics in each pixel, it is noteworthy that still there are distortions in the acquired image at different spatial frequencies. First, sensitivity variations (quantum efficiency nonuniformity) of certain pixels can cause this (Pimonov & Terebizh, 1981). Second, the distortions are caused by interference effects which occur on a thin layer of a substrate in a back-illuminated CCD (Lesser, 1990), so-called fringes. Their amplitude depends on wavelength and substrate thickness. To estimate the count statistics distortions, we obtained an image of a flat field with the EEV CCD42-90 detector in the spectral range of 760–920 nm. In this case, a root-mean-square amplitude of image modulation by fringes was approximately 2.6% and the modulation period was about 30 pixels. Then a sample of random series of centers $\{x_i, y_i\}$ of square fragments (*box*) with different sizes w and a volume of about 10^4 was generated. The Fano factor $k_i(w)$ was calculated in each fragment, and the selective Fano factor $\overline{k(w)}$ —over the whole sequence. Figure 2 shows the dependence of the selective Fano factor on the fragment size, which demonstrates the count statistics distortion at different spatial frequencies. The same figure demonstrates the values for a fringe-free image in accordance with (Howell, 2012).

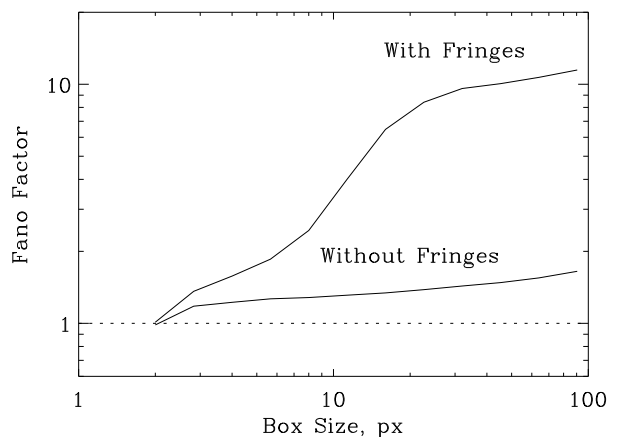


Fig. 2. Dependence of the selective Fano factor on the fragment size before and after the removal of fringes for CCD42-90. A dashed line indicates the value characteristic for a Poisson flux.

The fact that the Fano factor slightly increases in the fringe-free image gives evidence of small distortions of the count statistics.

3. COUNT STATISTICS DURING FAINT OBJECT DETECTION

Let us assume that the incoherent radiation flux $F = F_{\text{obj}} + F_{\text{sky}}$, which is the sum of fluxes from a studied object and the sky background and $F_{\text{obj}} \leq F_{\text{sky}}$, is falling on the input of the registration system by which we mean the system *atmosphere + telescope + spectrograph + CCD*. With exposures exceeding the time of atmospheric coherence, the influence of the latter can be neglected; and it is a quite true hypothesis that a flux at the registration system entrance is a random process with a Poisson distribution which becomes close to a Gaussian

distribution (Monin & Yaglom, 2007) in the case of great fluxes. Then, the measured signal at the output of the registration system at the point with coordinates (x, y) can be set with the expression:

$$N(x, y) = F(x, y)flat(x, y) + bias, \quad (5)$$

where $bias$ is a zero level of a CCD system and $flat(x, y)$ is a flat field function determining the variation of a CCD transfer function over the field of view. The $bias$ level is a random value and its root-mean-square error is readout noise of a CCD system.

Hereinafter, by faint objects we mean objects both with low flux and detectable against the sky background with some contrast c :

$$c = (F_{obj+sky} - F_{sky})/F_{sky}. \quad (6)$$

For example, the object fainter than the 20th magnitude in the visible range with typical images at the 6-m telescope ($1''.5-2''$) should have a contrast of 1.

During actual observations of faint objects, two independent values are measured: $N_{obj+sky}$ and N_{sky} , which are defined with the relations:

$$\begin{aligned} N_{obj+sky} &= (F_{obj} + F_{sky})flat(x, y) + bias_{obj+sky}, \\ N_{sky} &= F_{sky}flat(x, y) + bias_{sky}, \\ N_{obj} &= N_{obj+sky} - N_{sky}. \end{aligned} \quad (7)$$

As a matter of fact, these expressions connect an observed mathematical expectation of a number of counts of the studied object with mean measured values. Let us note that the transfer function $flat(x, y)$ in each observing run is determined with some error, thus, it is also a random value. Our laboratory measurements show that the realistic distribution of estimation errors of the flat field and readout noise of a readout channel are well presented with the Gaussian distribution.

The ultimate aim of observations is not only measuring mean values but also the reconstruction of signal statistical properties from the analysis of count statistics at the output of a readout channel which introduces distortions. Solving of this problem in general terms seems quite difficult and is not a goal of the present paper.

To estimate the statistics distortions in an existing CCD system, we carried out a numerical modeling with the Monte-Carlo technique. It was supposed that a random photon flux with a Poisson distribution fall on the CCD system input. It was also accepted that estimation errors of the flat field and readout noise have a Gaussian distribution. We studied the case of observation of the object with a contrast of $c = 1$ ($F_{obj} = F_{sky}$). The object's Fano factor for a random number sample of a volume of 10^4 instances was calculated at each sky background level.

Figure 3 shows the dependence of the Fano factor on the sky background level for different errors in the flat-field determination. Readout noise of $3e^-$ was accepted in calculations. As the figure indicates, the Fano factor is minimal at a certain level of the sky background, i.e.,

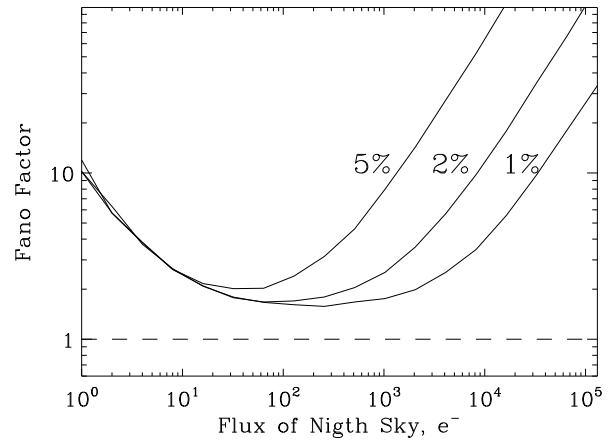


Fig. 3. Dependence of a Fano factor on a level of the sky background for different errors in the flat-field determination—1%, 2%, and 5%. A dashed line indicates the value characteristic for a Poisson flux.

in this case, the CCD system introduces minimum distortions of the counts statistics and, correspondingly, a signal-to-noise ratio in this region is maximum. The increasing of the Fano factor on low fluxes is obviously determined by the readout noise of a CCD readout channel.

Increasing of the Fano factor with the growth of the sky background brightness means that at a high signal level, the noise in an image is determined by uncertainty of a flat field value. Consequently, a signal-to-noise ratio of a measured object does not grow with the increase of a sky background flux. It is shown in Fig. 4, which demonstrates the result obtained with a numerical modeling using another units. This result is important to get the maximum S/N ratio during observations of faint objects using actual CCD detector systems with a readout noise, and in an inhomogeneous sensitivity correction procedure is used. It is noteworthy that an error in the flat field determination depends not only on a signal level of calibration images but also on the presence of residual noise after removal of fringes, unavoidable dust particles on the optical elements of a registration system (an focal reducer or a spectrograph), and on the violation of telecentric conditions in a calibration path, etc. The influence of the last factor can be essentially reduced by using images or spectra of twilight sky for calibration.

Dependences shown in Fig. 3 allow us to set the limiting values of optimal exposures for observations with different modes of the SCORPIO multipurpose spectrograph (Afanasiev & Moiseev, 2005), at which the CCD systems with EEV CCD42-40 and CCD42-90 detectors, are used as registration systems. For instance, in the direct image mode, a flux from the moonless sky background greater than $100e^-$ is achieved in the green spectral region (the V filter) in 10-second exposure and in the red one (the R filter)—in 5-second exposure; and as it follows from Fig. 3, minimum distortions of the counts statistics and maximum signal-to-noise ratio are achieved in the flux

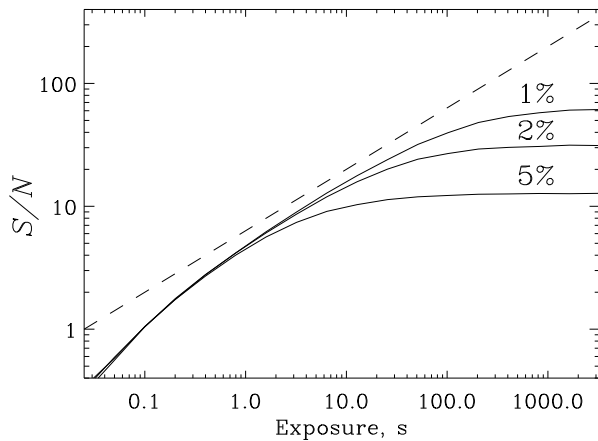


Fig. 4. Signal-to-noise ratio for an object depending on exposure for different errors in the flat-field determination—1, 2, and 5%. A dashed line indicates the S/N value for a Poisson flux.

range of 10^2 – $10^3 e^-$. As can be seen from Fig. 4, with exposures greater than 100 s, the S/N ratio does not almost increase. In order to extend the detection threshold, the most correct observational approach is not the extension of an exposure time but increasing a number of short-term exposures. After independent processing of each image, they can be combined and the signal-to-noise ratio will $\sqrt{N_{\text{exp}}}$ times grow. It is necessary that flat fields of each exposure do not coincide, which corresponds to image acquisition with an object shift or observations in different nights. Otherwise, an inhomogeneity value ceases to be random and its contribution does not decrease after combining images. It should be noted that such a way of observations not only increases the S/N but also allows us to efficiently remove cosmic rays in the combined image if the number of exposures is more than three. In this case, a median and robust average are calculated in each channel. Such an algorithm helps to eliminate cosmic rays without distorting the spectrum of spatial image frequencies.

4. CONCLUSIONS

For the first time a statistics of counts at the output of a CCD system has been studied using measurements of a Fano factor. Distortions of statistics when acquiring faint objects has also been investigated. It is shown that:

- (1) For the existing CCD readout channel designed in the Advanced Design Laboratory, the deviation from the Poisson statistics is observed for fluxes weaker than $100 e^-$ with a readout noise of $3 e^-$. The CCD system can be regarded as almost “ideal” in the flux range of 10^2 – $10^4 e^-$.
- (2) Distortions of counts statistics at various spatial frequencies are small—the Fano factor increases 1.5 times at scales of 10 to 100 pixels after the removal of fringes.

- (3) The Fano factor for the counts of a faint object with a contrast of about 1 depends on the sky background level, and the increase of exposure time does not cause the increase of a signal-to-noise ratio due to errors in the flat field determination.

The results derived can be used to choose optimum exposure ranges in detecting extremely low signals when one is registering background radiation and spectra of distant faint galaxies.

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